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New Deformation of Para-Bose Statistics

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We propose commutation relations for a single mode $\hat{g}$-deformed para-Bose oscillator. In this new deformation of para-Bose statistics the distribution function has the same form as in the para-Bose statistics. Furthermore, we show analogies between the coherent states of $\hat{g}$-deformed and q-deformed para-Bose statistics.

KEY WORDS: deformation; para-Bose statistics.

1. INTRODUCTION

Recently much effort has been devoted to the study of deformed structures, both in the context of quantum group and of Lie-admissible algebras. It is known that the concept of intermediate statistics is not new, it dates back to the 1950s. Since the work by Wilczek (1982a,b), it has been studied extensively and was found to be useful to study fractional quantum Hall effect (Halperin, 1984) and superconductivity (Langhlin, 1988).

One of the possible approaches to intermediate statistics consists in deforming the bilinear Bose and Fermi commutation relations. Particles which obey this type of statistics are called quons (Greenberg, 1991). If we consider a system with a single degree of freedom, we obtain the relation $aa^{+} - qa^{+}a = 1$, i.e., the q-oscillators. It was first introduced by Arik and Coon (1976) and Kuryshkin (1980), and later rediscovered in the context of quantum SU (2) (Biedenharn, 1989; Macfarlane, 1989; Woronowicz, 1987). Recently, a version of fractional statistics has been proposed which possesses some operational characteristies. In this type of approach, the c-number q which appears in the q-deformed algebras is replaced by an operator $\hat{g}$ which gives the generalized statistics interesting properties (De Falco *et al.*, 1995a,b,c; De Falco and Mignani, 1996; Scipioni, 1993a,b, 1994; Wu *et al.*, 1992; Zhao *et al.*, 1995).

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It is known (Green, 1953; Greenberg, 1990; Ohnuki and Kamefuchi, 1982) that besides the ordinary Bose and Fermi statistics there exist their para-Bose and para-Fermi generalizations. Chaturvedi and Srinivasan (1991) showed that a single para-Bose oscillator may be regarded as a deformed Bose oscillator. The commutation relations (CR) for a single mode of the harmonic oscillator which contains para-Bose and q-deformed oscillator CR are constructed (Krishma Kumari, 1992). Next, the connection of q-deformed and generalized deformed para-Bose oscillators with para-Bose oscillators has been determined (Bang, 1994, 1996), and some properties of the deformed para-Bose systems have also been considered (Bang, 1995a,b; Bang and Mansur Chowdhury, 1997; Chakrabarti and Jagannathan, 1994; Shanta *et al.*, 1994).

Naturally, the following question can be raised: how can the $\hat{g}$-deformed commutation relations for a single-mode para-Bose oscillator be generalized in the case of $\hat{g}$-deformation.

The main purpose of this work is devoted to this question. In addition, we discuss the distribution function and the coherent states of the annihilation operators corresponding to $\hat{g}$-deformed para-Bose oscillators.

2. $\hat{g}$-DEFORMED PARA-BOSE OSCILLATORS

As is well known, a single mode para-Bose system (Bang, 1996; Chaturvedi and Srinisavan, 1991) is characterized by the CR

$$[a, \mathcal{N}] = a, \quad [a^+, \mathcal{N}] = -a^+ \tag{1}$$

where

$$\mathcal{N} = \frac{1}{2}(aa^+ + a^+a) = \frac{p}{2} \tag{2}$$

and p is the order of the para-Bose system. Also

$$aa^+ = f(\mathcal{N} + 1), \quad a^+a = f(\mathcal{N}) \tag{3}$$

with

$$f(n) = n + \frac{1}{2}\{1 - (-1)^n\}(p - 1). \tag{4}$$

Hence

$$[a, a^+] = g(\mathcal{N}) \tag{5}$$

where

$$g(\mathcal{N}) = f(\mathcal{N} + 1) - f(\mathcal{N}) = 1 + (-1)^{\mathcal{N}}(p - 1). \tag{6}$$

From these relations, an operator A^+ was constructed so that (Chaturvedi and Srinisavan, 1991; Krishma Kumari *et al.*, 1992)

$$A^+ = a^+ \frac{\mathcal{N}+1}{f(\mathcal{N}+1)} \tag{7}$$

$$[a \cdot A^+] = 1, \quad [A^+, \mathcal{N}] = -A^+. \tag{8}$$

By the results just mentioned the number operator $\mathcal{N}$ can be written as

$$\mathcal{N} = A^+ a. \tag{9}$$

Let us now turn to the question of the $\hat{g}$-deformed para-Bose oscillator. According to the method in Krishma Kumari *et al.* (1992) we can $\hat{g}$-deform the para-Bose CR namely by proposing the CR to be

$$\tilde{a}\tilde{A}_g^+ - \hat{g}\tilde{A}_g^+\tilde{a} = 1, \tag{10}$$

where $\tilde{A}_g^+$ and N are given by

$$\tilde{A}_g^+ = \frac{\tilde{a}^+(N+1)}{f(N+1)}, \tag{11}$$

$$N = \tilde{A}_g^+\tilde{a}. \tag{12}$$

By using (11) and (12) we obtain

$$\tilde{a}^+\tilde{a} = f(N), \tag{13}$$

$$\tilde{a}\tilde{a}^+ = (\hat{g}N+1)\frac{f(N+1)}{N+1}. \tag{14}$$

With the help of (13) and (14) we get

$$\begin{aligned} [\tilde{a}, \tilde{a}^+] &= f(N+1) - f(N) + \hat{o}\frac{N}{N+1} f(N+1) \\ &\equiv h(N), \end{aligned} \tag{15}$$

where

$$\hat{o} = \hat{g} - 1. \tag{16}$$

In so doing, we are led to the CR for $\hat{g}$-deformed para-Bose oscillators.

3. STATISTICAL DISTRIBUTION

Consider now the $\hat{g}$-deformed Green function defined as the statistical distribution of $\tilde{a}^+\tilde{a}$. The statistical distribution of the operator F is defined through the formula:

$$\langle F \rangle = \frac{1}{Z}\mathrm{tr}(e^{-\beta H} F), \tag{17}$$

where Z is the partition function,

$$Z = \mathrm{tr}(e^{-\beta H}), \tag{18}$$

which determines the thermodynamic properties of the system, $\beta = \frac{1}{kT}$, H is Hamiltonian, which is usually taken of the form $H = \omega N$, ω being one particle-oscillator energy. The trace must be taken over a complete set of states.

It follows readily from (17), (18), (4), and (13) that

$$Z = \frac{e^{\beta\omega}}{e^{\beta\omega} - 1}, \tag{19}$$

$$\mathrm{Tr}(e^{-\beta H}\tilde{a}^{+}\tilde{a}) = \frac{e^{\beta\omega}(pe^{\beta\omega} - p + 2)}{(e^{\beta\omega} - 1)^2(e^{\beta\omega} + 1)}. \tag{20}$$

Hence,

$$\langle \tilde{a}^{+}\tilde{a} \rangle = \frac{pe^{\beta\omega} - p + 2}{e^{2\beta\omega} - 1}. \tag{21}$$

We would like to note here that the formulae (19)–(21) coincide exactly with the corresponding ones of the para-Bose statistics.

4. COHERENT STATES

In the last part of the paper we consider the construction of coherent states of the annihilation $\tilde{a}$, that is

$$\tilde{a}|z\rangle = z|z\rangle, \tag{22}$$

where z is a complex number.

The construction of these coherent states is most easily done following a simple technique (Bang, 1976; Chaturvedi and Srinisavan, 1991; Shanta *et al.*, 1994) applicable to any generalized boson oscillator. The coherent states have the form

$$|z\rangle \sim \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{N_n}}|n\rangle, \tag{23}$$

where

$$N_n = \langle 0|\tilde{a}^n \tilde{a}^{+n}|0\rangle. \tag{24}$$

Using (15), it is not difficult to prove that

$$N_n = \{n\}!, \tag{25}$$

with

$$\{n\} = h(n-1) + \cdots + h(0). \tag{26}$$

Finally, the normalized coherent state is

$$\begin{aligned} |z\rangle &= \left(\sum_{n=0}^{\infty} \frac{|z|^{2n}}{\{n\}!}\right)^{-1/2} \sum_{n=0}^{\infty} \frac{z^n}{\sqrt{\{n\}!}} |n\rangle \\ &= \left(\sum_{n=0}^{\infty} \frac{|z|^{2n}}{\{n\}!}\right)^{-1/2} \exp_g(z\tilde{a}^+)|0\rangle, \end{aligned} \tag{27}$$

where $\exp_g(x)$ is defined as

$$\exp_g(x) \equiv \sum_{n=0}^{\infty} \frac{x^n}{\{n\}!}. \tag{28}$$

It is worth noticing that this form of coherent states are similar to the known relations of the coherent states in the case of q-deformed para-Bose statistics (Bang, 1996; Chakrabarti and Jagannathan, 1994, Shanta *et al.*, 1994).

5. CONCLUSIONS

To conclude, we have proposed the $\hat{g}$-deformed CRs for a single mode para-Bose oscillator and have constructed the distribution function and coherent states for this deformation of para-Bose statistics. We think that our method will also be applied to the case of para-supersymmetry.

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